***Introduction to Measurement Theory***

Measure on a set is a systematic way to assign numbers to suitable subsets of that set, it is a generalization of Length, Area and Volume.

A Measure is a function that assigns a non-negative real number to a subset of set X, it must assign 0 to the empty set and be countably additive.

REMINDER: LIMITS OF SETS

Assume the sequence

Algebra on a set:

If Ω is a set and F = {Subsets of Ω: F ≤ P (Ω)} then F is an **algebra** if and only if

* If A
* If A,B

If Ω is a set and F = {Subsets of Ω: F ≤ P (Ω)} then F is a **Sigma-Algebra** if and only if

* F is an algebra
* If

A **Sigma-Algebra F** also forces the following properties:

* Ω

**Sigma-Algebra** generated by an arbitrary family

Let **S** be an arbitrary family of subsets of Ω such that then there exists a unique smallest Sigma-Algebra that contains all the sets of S (even though S may or may not be a sigma algebra) this -algebra is denoted and is the intersection of all -algebras that contain S.

This -algebra is called: The -algebra generated by S

**Sigma-Algebra** generated by a function

Let f be a function from a set X to a set Y and B is a σ-algebra of subsets of Y, then the σ-algebra generated by the function f, denoted by σ(f), is the collection of all inverse images  (S) of the sets S in B. i.e.

σ(f) = (S) | S

A function f from a set X to a set Y is [measurable](https://en.wikipedia.org/wiki/Measurable_function) with respect to a σ-algebra Σ of subsets of X if and only if σ(f) is a subset of Σ

**Topological Space:**

An ordered pair (α, β) where α is a set and β is a collection of subsets of α with the following properties:

* β
* β (Any UNION, finite or infinite belongs to β)
* β (Finite INTERSECTION belongs to β)

Using DE-MORGAN law's this definition can be changed to "CLOSED SET DEFINITION"

**Measurements**

Measurable Space is the double (Ω, F) where

Measure on a Measurable space is a function

[µ:

* µ(

if µ(Then µ is a finite measure, if µ(

**Measure Space / Probability Space:**

Measure space is a triple ( µ) where (µ is a Measure on ( If µ is a Probability Measure Then ( µ) is a Probability Space.

**Properties:**

* **MONOTONITY:**

µ(

* **SUB-ADDTIVITY:**

* **CONTINUNITY FROM BELOW:**

Let

* **CONTINUNITY FROM ABOVE:**

Let